Table 1 Approximate solution compared with Graham's results

		Equation (8)		Graham[8]	
k_{xo}	k_{yo}	Amplitude	Phase	Amplitude	Phase
0	0	1	0	1	0
0	0.1	0.8546	0		
0	0.25	0.6752	0	0.6762	0
0	0.5	0.4559	0	0.4869	0
0	1.0	0.2079	0	0.2970	0
0.25	0	0.6744	−12.3	0.6744	−12.3
0.25	0.1	0.6543	−10.0		
0.25	0.25	0.5732	−4.4	0.5930	−4.9
0.25	0.5	0.4151	3.1	0.4623	1.6
0.25	1.0	0.1978	12.2	0.2925	7.4
0.5	0	0.5265	−4.8	0.5265	−4.8
0.5	0.1	0.5184	−3.2		
0.5	0.25	0.4799	1.7	0.4899	0.2
0.5	0.5	0.3803	11.1	0.4139	7.0
0.5	1.0	0.1994	26.0	0.2809	15.5
1.0	0	0.3896	18.9	0.3896	18.9
1.0	0.1	0.3865	19.9		
1.0	0.25	0.3712	23.4	0.3706	21.6
1.0	0.5	0.3236	31.9	0.3322	26.3
1.0	1.0	0.2032	50.6	0.2499	34.9
2.0	0	0.2801	73.1	0.2801	73.1
2.0	0.1	0.2790	73.7		
2.0	0.25	0.2734	76.0	~0.268	~ 76.0
2.0	0.5	0.2543	82.2	0.2460	77.1
2.0	1.0	0.1933	99.2	0.1989	83.0

and P_∞ is given by Eq. (3), with $M_\infty = 0$, i.e., the Sears gust solution. Thus, for incompressible flow the change in the airfoil response produced by skewing the gust is to multiply the nonskewed result by the factor exp $[ik_{x0}g(k_{y0})/k_{x0})]$ when k_{y0} is small.

Since gk_{x0} is independent of x, the relations for lift and moment also are multiplied by the same factor. Thus, the normalized lift is

$$\tilde{L} = S(k_{x0})\exp[ik_{x0}g(k_{y0}/k_{x0})] \tag{8}$$

Since the quarter-chord moment for the Sears gust case is zero, it is zero for the present solution also. According to the numerical results of Graham,[8] and also the approximate solution for large k_{y0} given in Refs. 1 and 2, the center of pressure does move forward from the quarter-chord point as k_{y0} increases. For small k_{y0}, however, this effect is not particularly important. The accuracy of the parallel gust result, Eq. (3), is $O(k_{x\infty}M_\infty/\beta_\infty^2)$. Thus, from Eqs. (2) the accuracy of the present skewed gust solution is $O(k_{y0})$. The forward movement of the center of pressure evidently must be $O(k_{y0}^2)$ or higher, since it is not given by the present solution.

Values calculated using Eq. (8) are compared in Table 1 with the numerical results of Graham.[8] The present approximate solution agrees favorably with the numerical results, especially considering the fact that the solution for large k_{y0} is shown in Ref. 2 to give accurate results for $k_{y0} > 0.25$. Thus, only the range $0 < k_{y0} < 0.25$ need be covered by the present solution.

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Technical Comments

Comment on "On Transient Cylindrical Surface Heat Flux Predicted from Interior Temperature Response"

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RECENTLY, Chen and Thomsen[1] published a note which purportedly presented an alternative solution for the inverse problem in hollow cylinders. Since the method is based upon the temperature inversion measured by *only one interior probe*, whereas previous investigations[3-4] required two imputs, I believe that a number of comments are in order. Due to the form of the differential equation, the linear inverse problem requires a minimum of three inputs: the initial condition and two others. The latter represent the experimental data taken from embedded sensors which can be either: temperature traces from the thermocouples positioned at two different locations, temperature and heat flux information gathered at one location, or the temperature data coupled with a boundary condition dictated by geometrical-thermal considerations. For the latter, Sparrow[2] derives an inversion method for a cylinder, which is solid, hence the technique is applicable for the thermally symmetric situation. In the hollow cylinder configuration, there is generally no thermal symmetry; consequently another method of analysis must be considered. Imber[4] presents a general extrapolation method for this important experimental situation, which utilizes only the experimental data from two thermocouples. Due to the presence of the modified Bessel functions, a short time solution is generated for temperature prediction in any direction, and numerical results are presented to demonstrate the effectiveness of the procedure.

It should be pointed out that Chen's analytical approach is remarkably similar to Imber's, since it incorporates his concept of exponential alteration or suppression (see Eq. (10)) as well as the methodology for the derivation of the solution. Furthermore, the use of Eq. (10) can present difficulties that the authors should be aware of. For small time, the values for the higher order repeated error integral functions decrease rapidly, accordingly the matrix used to generate the coefficients b_n can be nearly singular when the number of

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equations N is too large. Practical experience indicates that the reported nineteen time imputs used by the authors is beyond the safe margin.

Lastly, the model chosen by the authors is suspect, since a finite hollow tube is replaced by a circular hole in an infinite plate. Solutions to the inverse problem indicate that the temperature extrapolation is sensitive. Under the circumstances indicated, a hollow finite walled tube with the outer face regarded as insulated or at a constant temperature would have been more suitable. It would have been very useful had the authors presented a comparison of these models with theirs. Finally, the authors' citation of Ref. 8 is incorrect as it appears. It should be Imber, M. and Khan, J.

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[1]Chen, C.J. and Thomsen, D.M., "On Transient Cylindrical Surface Heat Flux Predicted from Interior Temperature Response," *AIAA Journal,* Vol. 13, May 1975, pp. 697-699.

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Reply by Author to M. Imber

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We would like to thank Professor Imber for taking the interest to comment on our paper. We were well aware of Professor Imber and Dr. Khan's work when we did our project.

We would like first to clarify what we meant by "only one interior probe." By this we mean that in the actual experimental measurement of heat flux to the inner suface of a hollow cylinder, only one probe is needed interior to the cylinder wall. We know that for the problem, the partial differential equation, Eq. (1) of our paper (Ref. 1), requires one initial and two boundary conditions, Eqs. (2-4) of Ref. 1. For our problem, we considered that the initial temperature distribution is uniform and that the two boundary temperatures are given by an interior probe and a probe at the outer surface. For a short time duration in which the temperature at the outer surface does not change we may replace the outer boundary by a larger radius. A practical situation of this type is the gun barrel heating problem where the duration of a shot is only 2 to 5 milliseconds while the temperature at the barrel outer surface does not change for 5 to 10 seconds after the firing.

The reason that we favor only one interior probe is that in practice any probes embedded interior to the cylinder will distort the temperature field and hence create errors in measurement.[2-4] On the other hand, at least one probe must be embedded interior to the cylinder wall to improve the response time and the magnitude of response which is important in the inversion problem. Therefore we conducted all our experiments with only one interior probe and the other probe was located at the outer surface. When the outer probe did not respond we found that we could not apply your Eqs. (8) and (7) (of your Ref. 4) for backward extrapolation. This is why we performed our analysis. Therefore for relatively short times our solution should apply satisfactorily and your analysis may apply only when both probes start to respond.

You are right in pointing out that the matrix associated with the use of our polynomial, Eq. (10), can be nearly singular when the number of equations, N, is too large. We had in our computer code a maximum of $N=20$ with good results. We found that a ten term polynomial is sufficient to give satisfactory results and use of N more than 20 will probably make the inversion of the matrix difficult because of increasing singular behavior as you mentioned.

In all our calculations we always checked the predicted temperature at the outer surface to validate our assumption of constant temperature. Finally we apologize for the typing error in Dr. Khan's name.

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Comment on "Boundary-Layer Transition on Supersonic Cones in an Aeroballistic Range"

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Potter[1] recently reported on a study of boundary-layer transition on supersonic cones in an aeroballistic range. A conclusion drawn from this investigation is that the local Reynolds number of transition increases with increasing unit Reynolds number. It is the purpose of this Note to show that measurements of the Reynolds number at which full turbulence occurs in the wake of high-speed cones shows a similar variation. The results of two aeroballistic range investigations of transition in the wake[2,3] are compared with Potter's results in Fig. 1. The subscript δ denotes computed body surface or "edge-of-boundary-layer" flow conditions, and the length characterizing transition is axial distance measured from cone apex.The good agreement between these wake and body transition measurements when compared in this manner suggests that they may be related phenomena. Similar effects are present in other aeroballistic range measurements of transition Reynolds numbers.[4-7]

In some of the attempts to correlate wake transition measurements[2-4,7] a Reynolds number based on body diameter has been used as a correlating parameter. A con-

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